Exercise 29

Refer to the law of laminar flow given in Example 7. Consider a blood vessel with radius 0.01 cm, length 3 cm, pressure difference 3000 dynes/cm^2, and viscosity $\eta = 0.027$.

(a) Find the velocity of the blood along the centerline $r = 0$, at radius $r = 0.005$ cm, and at the wall $r = R = 0.01$ cm.

(b) Find the velocity gradient at $r = 0$, $r = 0.005$, and $r = 0.01$.

(c) Where is the velocity the greatest? Where is the velocity changing most?

[TYPO: Replace "$\eta = 0.027$" with "$\eta = 0.027$ poises," replace "$r = 0.005$" with "$r = 0.005$ cm," and replace "$r = 0.01$" with "$r = 0.01$ cm".]

Solution

Part (a)

According to the law of laminar flow on page 230,

$$v = \frac{P}{4\eta l}(R^2 - r^2).$$

For a blood vessel with radius 0.01 cm, length 3 cm, pressure difference 3000 dynes/cm^2 = 3000 g/(cm $\cdot$ s^2), and viscosity $\eta = 0.027$ poises = 0.027 g/(cm $\cdot$ s), it becomes

$$v = \frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{4\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}[(0.01\text{ cm})^2 - r^2].$$

If $r = 0$, then

$$v(0) = \frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{4\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}[(0.01\text{ cm})^2 - 0^2] \approx 0.925926\ \frac{\text{cm}}{\text{s}}.$$

If $r = 0.005$ cm, then

$$v(0.005) = \frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{4\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}[(0.01\text{ cm})^2 - (0.005\text{ cm})^2] \approx 0.694444\ \frac{\text{cm}}{\text{s}}.$$

If $r = 0.01$ cm, then

$$v(0.01) = \frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{4\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}[(0.01\text{ cm})^2 - (0.01\text{ cm})^2] = 0\ \frac{\text{cm}}{\text{s}}.$$

Part (b)

Take the derivative of the velocity function with respect to r to obtain the velocity gradient.

$$\frac{dv}{dr} = \frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{4\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}(-2r) = -\frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{2\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}r$$

If $r = 0$, then

$$\frac{dv}{dr}(0) = -\frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{2\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}(0) = 0\ \frac{\text{cm/s}}{\text{cm}}.$$

If $r = 0.005$ cm, then

$$\frac{dv}{dr}(0.005) = -\frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{2\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}(0.005\text{ cm}) \approx -92.5926\ \frac{\text{cm/s}}{\text{cm}}.$$

If $r = 0.01$ cm, then

$$\frac{dv}{dr}(0.01) = -\frac{3000\ \frac{\text{g}}{\text{cm}\cdot\text{s}^2}}{2\left(0.027\ \frac{\text{g}}{\text{cm}\cdot\text{s}}\right)(3\text{ cm})}(0.01\text{ cm}) \approx -185.185\ \frac{\text{cm/s}}{\text{cm}}.$$

Part (c)

The velocity is the greatest at $r = 0$, and the velocity is changing the most at $r = 0.01$ cm.